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A note on rotating Hele Shaw cells

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The effect of rotation on the flow around an obstacle in a Hele Shaw cell is considered. For the flow outside the strip-shaped boundary layer near the obstacle, it is shown that the rotating and non-rotating cases are related by a simple transformation. The flow in the boundary layer near the obstacle is computed numerically and its relation to the Stewartson $E^{\frac{1}{4}}$ and $E^{\frac{1}{3}}$ layers is briefly discussed.

1. Introduction

Flows in Hele Shaw cells, i.e. flows of viscous fluids between two large and closely spaced parallel plates, are described in almost every text-book on elementary fluid mechanics. If the flow between the plates is locally blocked by an obstacle whose length scales in the directions parallel to the plates are significantly larger than the distance between the plates, it was shown both experimentally and theoretically by Hele Shaw (1898) that, apart from thin strip-shaped boundary layers near the edges of the plates and the obstacle, the streamlines are parallel to the plates and have the same geometry as those of a two-dimensional potential flow. The variation of the velocity field in the direction perpendicular to the plates is, to a high degree of accuracy, locally the same as that of a plane Poiseuille flow. The case when the plates are rotating around an axis, which is not parallel to the plates, was studied by Goldin (1957) for source–sink flows. Goldin showed that the local plane Poiseuille flow in the non-rotating case is in the rotating case replaced by two merging non-divergent Ekman layers. If the distance between the plates is large, the Ekman layers will, of course, be separated by a geostrophic flow.

In the present note, the modification of the flow between rotating plates caused by an obstacle between the plates is considered. It is shown that, although the structure of the boundary layer near the obstacle becomes considerably more complicated than in the non-rotating case, there is a simple transformation between the non-rotating and rotating cases for the flow outside the boundary layer. The boundary-layer problem considered in this work is a simpler version of the boundary-layer problem solved by Hashimoto & Matsuda (1979) for the motion of an ideal viscous gas in the neighbourhood of the edge of a rapidly rotating cylinder, whose axial dimension is much smaller than its radial dimension.

It may be of some interest to note that flows of the kind considered in the present work appear in some technological applications. One important case is the flow between the disks in the very densely packed disk stacks in centrifugal separators for separation

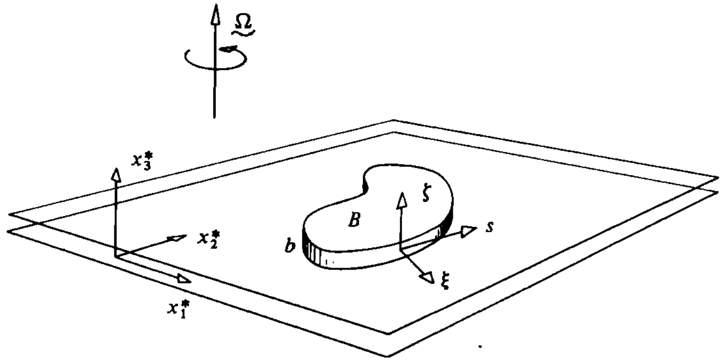


FIGURE 1. Sketch of the geometrical configuration and definition of co-ordinate systems.

of fluid emulsions or suspensions. Such disk stacks do significantly increase the efficiency of centrifugal separators (Sokolov 1971). Another application is the friction pump, where the radial transport of fluid between two rotating plates is driven by friction and centrifugal forces. In both the aforementioned applications, there are usually caulks (or similar devices) between the plates and the effect of these on the flow is of some interest.

The paper is disposed as follows. Section 2 contains the mathematical formulation of the problem and the solution for the flow outside the boundary layer. Some numerical solutions of the boundary-layer problem, obtained by a Galerkin method, are given in §3.

2. The flow outside the boundary layer

Consider two infinitely large parallel plates, which are rotating around a common axis with the angular velocity Ω. The axis of rotation is, for simplicity, assumed to be perpendicular to the plates. A Cartesian co-ordinate system (x_1^*, x_2^*, x_3^*), where asterisks denote dimensional co-ordinates, will be used. The co-ordinate system is rotating with the plates. The x_3^* direction is taken to be parallel to the axis of rotation, see figure 1. The distance between the plates is $2\delta L$, where δ is a dimensionless quantity and L is a characteristic length, which is specified below. The origin of the co-ordinate system is chosen such that the plates are at $x_3^* = \pm\delta L$. Between the plates there is a body B (see figure 1), which is bounded by the plates at $x_3^* = \pm\delta L$ and by the cylindrical surface $g(x_1^*/L, x_2^*/L) = 0$. The latter equation defines the length scale L. The surface $g = 0$ will in what follows be called b.

The space between the plates outside B contains a Newtonian fluid, whose density ρ and kinematic viscosity ν are assumed to be constants. The motion of the fluid relative to the plates is assumed to be driven by prescribed sources and sinks and/or a prescribed steady pressure field at large distances from B. The order of magnitude of the velocity of the fluid relative to the plates is given by a characteristic velocity U.

The flow is suitably characterized by the following non-dimensional variables:

$$\text{Rossby number,}\quad Ro = \frac{U}{L\Omega};$$

$$\text{Ekman number,}\quad E = \frac{\nu}{L^2\Omega};$$

Taylor number, $T = \frac{\delta^2}{E}$.

The Rossby number is assumed to be sufficiently small for linear theory to be valid. The Ekman number is also assumed to be small whereas the Taylor number is assumed to be of order unity, i.e. $\delta = O(E^{\frac{1}{2}})$. Non-dimensional variables are defined as

$$\mathbf{x} = \frac{\mathbf{x}^*}{L},$$

$$\mathbf{u} = (u, v, w) = \frac{\mathbf{u}^*}{Ro\, U} \quad \text{(velocity)},$$

$$p = \frac{p^*}{Ro\rho UL\Omega} \quad \text{(pressure)}.$$

The linearized equations for $\mathbf{u}$ and p are

$$2\mathbf{k} \times \mathbf{u} = -\nabla p - E\nabla \times \nabla \times \mathbf{u}, \quad \nabla . \mathbf{u} = 0, \tag{2.1), (2.2}$$

where $\mathbf{k} = (0, 0, 1)$. Equations (2.1)–(2.2) are to be solved subject to the following boundary conditions

$$\mathbf{u} = 0 \quad \text{for } x_1, x_2 \text{ outside } B, \quad x_3 = \pm\delta; \tag{2.3a}$$

$$\mathbf{u} = 0 \quad \text{for} \quad g = 0, \quad |x_3| \leqslant \delta. \tag{2.3b}$$

The boundary conditions at large distances from B and in the neighbourhood of eventual singularities need not be specified at present. Because $\delta = O(E^{\frac{1}{2}})$, it is natural to use a stretched variable ζ, which is defined as

$$\zeta = \frac{x_3}{E^{\frac{1}{2}}}, \quad |\zeta| \leqslant T^{\frac{1}{2}}$$

and assume that the solution of (2.1)–(2.2) possesses an expansion of the form

$$[\mathbf{u}, p] = \sum_{l=0}^{N-1} E^{\frac{1}{2}l} [\mathbf{u}_l(\mathbf{y}, \zeta), p_l(\mathbf{y}, \zeta)] + O(E^{\frac{1}{2}N}), \tag{2.4}$$

where $\mathbf{y} = (x_1, x_2)$. In terms of the complex variable $z = x_1 + ix_2$, it follows from (2.1)–(2.2), (2.3a) and (2.4) that

$$w_0 = w_1 = 0, \quad u_0 - iv_0 = \frac{d\Pi_0}{dz} h(\zeta), \tag{2.5}$$

$$p_0 = \mathscr{R}[\Pi_0(z)], \quad h(\zeta) = -\frac{i}{2}\left(1 - \frac{\cosh(1-i)\zeta}{\cosh(1-i)T^{\frac{1}{2}}}\right),$$

where Π_0 is an analytic function of z. A solution of the form (2.5) was given by Goldin (1957) for source flows. When T is small, i.e. when the Coriolis force is weak compared with the viscous forces, equation (2.5) reduces, of course, to the classical solution given by Hele Shaw (1898). The remaining part of the problem is to determine the boundary condition for the function Π_0 on the curve $g(x_1, x_2) = 0$. The solution (2.5) cannot satisfy (2.3b) unless Π_0 = constant, which means that there is no flow anywhere. There will thus be a boundary layer on b and the boundary condition for Π_0 is determined by considering both (2.5) and the appropriate boundary-layer solution. If $\Omega = 0$, one finds after some simple calculations the following boundary condition

$$\mathbf{u}_0 . \mathbf{e}_n = 0 \quad \text{on } b, \quad \mathbf{e}_n = \pm \frac{\nabla g}{|\nabla g|} \tag{2.6a,b}$$

as was postulated on heuristic grounds by Hele Shaw (1898). If $\Omega \neq 0$, (2.6*a*, *b*) give

$$\{\mathbf{e}_n . \nabla p_0\} \mathscr{R}[h(\zeta)] + \{(\mathbf{k} \times \mathbf{e}_n) . \nabla p_0\} \mathscr{I}[h(\zeta)] = 0 \quad \text{on } b. \tag{2.6c}$$

Because $\mathscr{R}[h] \neq \mathscr{I}[h] \neq 0$ for $|\zeta| \leqslant T^{\frac{1}{2}}$, except possibly at isolated points, (2.6*c*) means that both the normal and the tangential derivatives of p_0 are zero on b, i.e. Π_0 is a constant. Equation (2.6*a*) is therefore obviously incorrect in the rotating case. (This awkwardness does not arise in the non-rotating case where $\mathscr{I}[h] \equiv 0$.) In order to determine the boundary condition for Π_0 on $g = 0$, there is thus a need to examine the boundary layer on b in some detail.

To describe the boundary layer, a local Cartesian co-ordinate system on b will be used (see figure 1). Expressed in the original co-ordinate system, the origin of the local co-ordinate system is at $\mathbf{x}_0 = (x_1(s), x_2(s), 0)$, where s is the arc length along the curve $g(x_1(s), x_2(s)) = 0$. In the local co-ordinate system, an additional stretched variable ξ is defined as

$$\xi = \frac{(\mathbf{x} - \mathbf{x}_0) . \mathbf{e}_n}{E^{\frac{1}{2}}}, \tag{2.7}$$

where the sign in (2.6*b*) is chosen such that $\mathbf{e}_n$ points outwards from B.

The correction velocity field in the boundary layer is written as

$$\hat{\mathbf{u}} = \hat{u}\mathbf{e}_n + \hat{v}(\mathbf{k} \times \mathbf{e}_n) + \hat{w}\mathbf{k}, \tag{2.8}$$

$\hat{\mathbf{u}}$ and the correction pressure field $\hat{p}$ are assumed to possess expansions of the form

$$[\hat{\mathbf{u}}, \hat{p}] = \sum_{l=0}^{N-1} E^{\frac{1}{2}l}[\hat{\mathbf{u}}_l(\xi, s, \zeta), \hat{p}_l(\xi, s, \zeta)] + O(E^{\frac{1}{2}N}). \tag{2.9}$$

Equations (2.1)–(2.2), (2.7) and (2.9) give the following lowest-order boundary-layer equations

$$\frac{\partial \hat{p}_0}{\partial \xi} = 0, \quad \frac{\partial \hat{p}_0}{\partial \zeta} = 0, \tag{2.10a}$$

$$-2\hat{v}_0 = -\frac{\partial \hat{p}_1}{\partial \xi} + \nabla_\xi^2 \hat{u}_0, \quad 2\hat{u}_0 = -\frac{\partial \hat{p}_0}{\partial s} + \nabla_\xi^2 \hat{v}_0, \tag{2.10b,c}$$

$$0 = -\frac{\partial \hat{p}_1}{\partial \zeta} + \nabla_\xi^2 \hat{w}_0, \quad \frac{\partial \hat{u}_0}{\partial \xi} + \frac{\partial \hat{w}_0}{\partial \zeta} = 0, \tag{2.10d,e}$$

where $\nabla_\xi^2 = \partial^2/\partial \xi^2 + \partial^2/\partial \zeta^2$. The solution of (2.10) has to fulfil the following boundary conditions:

$$\hat{\mathbf{u}}_0(\xi, s, \pm T^{\frac{1}{2}}) = 0, \tag{2.11a}$$

$$\left.\begin{aligned} \hat{u}_0(0, s, \zeta) &= -\mathbf{u}_0(s, \zeta) . \mathbf{e}_n(s) \\ \hat{v}_0(0, s, \zeta) &= -\mathbf{u}_0(s, \zeta) . (\mathbf{k} \times \mathbf{e}_n(s)) \\ \hat{w}_0(0, s, \zeta) &= 0 \end{aligned}\right\} \quad \text{for} \quad |\zeta| \leqslant T^{\frac{1}{2}}, \tag{2.11b,c,d}$$

$$\lim_{\xi \to \infty} (\hat{\mathbf{u}}_0, \hat{p}_0, \hat{p}_1) = 0, \tag{2.11e}$$

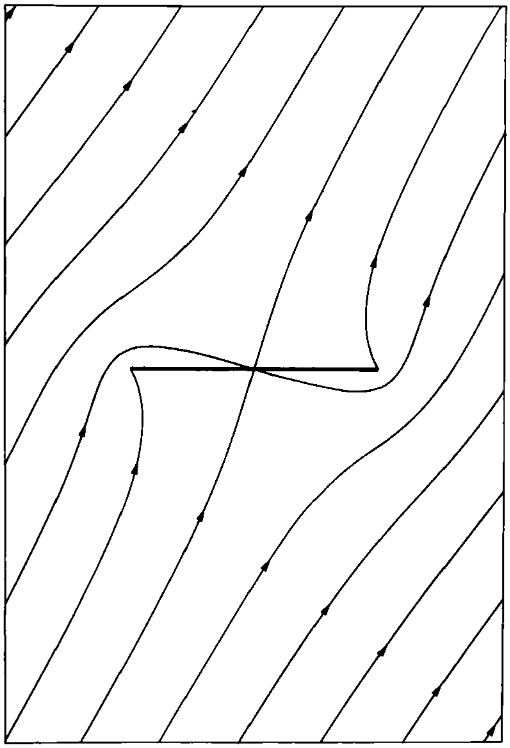


FIGURE 2. Flow around a flat plate of negligible thickness. The vertically averaged flow is at large distances perpendicular to the plate. $T = 25$, $\zeta = 4{\cdot}75$.

where $\mathbf{u}_0(s, \zeta)$ is the solution of (2.1)–(2.2) expressed in the original co-ordinate system and evaluated on b. If averages in the $\mathbf{k}$ direction are denoted by angle brackets one finds from (2.10*e*), (2.11*a*) and (2.11*e*) that

$$\langle \hat{u}_0 \rangle = \frac{1}{2T^{\frac{1}{2}}} \int_{-T^{\frac{1}{2}}}^{T} \left[\int_{\xi}^{\infty} \frac{\partial \hat{w}_0}{\partial \zeta}(\xi', s, \zeta')\, d\xi' \right] d\zeta' = 0. \tag{2.12}$$

Equations (2.12) and (2.11*b*) then give

$$\langle \mathbf{u}_0 . \mathbf{e}_n \rangle = 0 \quad \text{on } b \tag{2.13}$$

which is the boundary condition sought. Some solutions of (2.10*b*–*e*) are given in the next section.

Equation (2.13) means that the average in the $\mathbf{k}$ direction of the velocity field has the same geometry as if the flow is not rotating. If the complex potential W for the vertically averaged flow in the non-rotating case, which is defined by

$$\langle u_0 \rangle - i \langle v_0 \rangle = \frac{dW}{dz}, \tag{2.14}$$

is known for a given geometry of B and for a given distribution of singularities, say, one finds from (2.5) and (2.13)–(3.14) that the function Π_0 for the corresponding rotating case is given by

$$\Pi_0 = 2iW \bigg/ \left(1 - \frac{\tanh(1-i)T^{\frac{1}{2}}}{(1-i)T^{\frac{1}{2}}}\right). \tag{2.15}$$

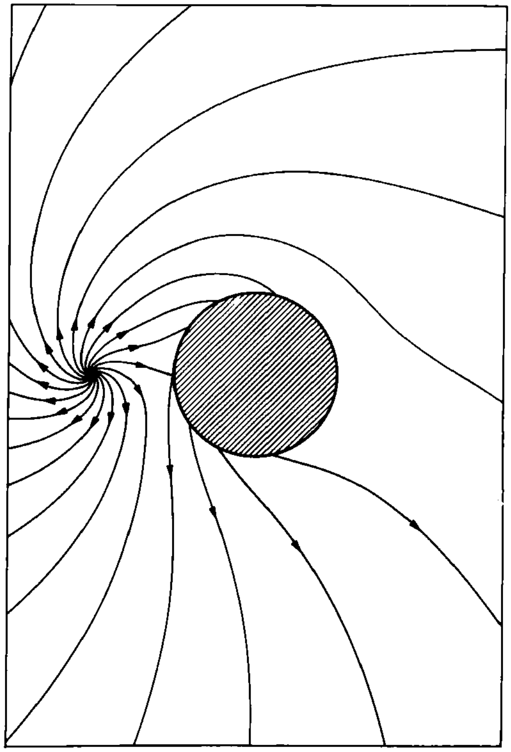


FIGURE 3. Source flow in the neighbourhood of a circular cylinder. $T = 25$, $\zeta = 4\cdot75$.

The dependence of the angle ϕ between the direction of the vertically averaged flow and the direction of the pressure gradient on the Taylor number T can be readily calculated from (2.14)–(2.15) whereby one finds the familiar limits

$$\lim_{T\to 0} \phi = 0, \quad \lim_{T\to\infty} \phi = \tfrac{1}{2}\pi.$$

It can be shown that the effect of rotation on the geometry of the streamlines (compared to the non-rotating case) outside the boundary layer is largest when T is large and $\pm T^{\frac{1}{4}} - \zeta = O(1)$, i.e. in the Ekman layers when these are distinct.

Two examples are shown in figures 2 and 3. Figure 2 shows the case when B is a flat plate of negligible thickness. It should be noted that one streamline goes into the boundary layer at the left edge and leaves the boundary layer at the right edge. The fluid particles on a streamline entering the boundary layer on one side of the plate are, of course, not the same as those on the same streamline leaving the boundary layer at the other side of the plate. The fluid particles are displaced in the $\mathbf{k}$ direction in the boundary layer. Figure 3 shows the flow from a source near a cylinder. Although the vertically averaged flow in the neighbourhood of the source is a potential source flow in the usual sense, the flow for each value of ζ is a potential source-vortex flow (cf. Goldin 1957).

3. The boundary-layer flow

It follows from (2.10*a*) and (2.11*e*) that $\hat{p}_0$ is zero. Elimination of $\hat{p}_1$ from (2.10*b–e*) then gives the following equation for $\hat{v}_0$

$$\left(\nabla_\xi^6 + 4\frac{\partial^2}{\partial \zeta^2}\right)\hat{v}_0 = 0, \tag{3.1}$$

which, with a different scaling of the co-ordinates, was studied by Stewartson (1957). In the present notation, Stewartson studied cases where $\delta = O(1)$, i.e. $T = O(E^{-1})$, and, by using asymptotic techniques for large values of T, discovered the set of boundary layers, which are known in the literature as the Stewartson $E^{\frac{1}{4}}$ and $E^{\frac{1}{3}}$ layers.

Following Stewartson (1957), a solution of (3.1) having the form

$$\hat{v}_0 = \sum_{K=1}^{3} C_K e^{-\omega\xi} \cosh \nu_K(\omega)\zeta, \quad \mathscr{R}[\omega] > 0 \tag{3.2}$$

is assumed. The quantities $\nu_K(\omega)$ are determined from (3.1). The homogeneous boundary conditions (2.11*a*) give three relations between the quantities $C_K(\omega)$ and do thus define an eigenvalue problem for ω. There are three sequences ω_n, one where ω_n is real and two where ω_n is complex, one sequence being the complex conjugate of the other. The asymptotic properties of the eigenvalues have been studied in some detail by Sundström & Bark (1980), who derived the following approximate relations

(i) $n \gg T^2$, T *arbitrary*

$$\omega_n = T^{-\frac{1}{2}}\left[l_n + \frac{T^2}{6l_n} + O\left(\frac{T^{\frac{8}{3}}}{n^{\frac{7}{3}}}\right)\right], \tag{3.3a}$$

$$\omega_n = T^{-\frac{1}{2}}\left[\frac{q_n}{4} - \frac{\ln q_n}{q_n} \pm \frac{i}{2}\ln q_n + O\left(\frac{2T^2+1}{n}\right)\right], \tag{3.3b}$$

where $$l_n = (n+\tfrac{1}{2})\pi, \quad q_n = (4n+1)\pi.$$

(ii) $T^{\frac{1}{2}} \ll n \ll T^2$, $T \gg 1$,

$$\omega_n = T^{-\frac{1}{2}}\left[r_n\{1+2s_n(1-s_n+2s_n^2)\} - \sqrt{\tfrac{3}{2}}\, s_n\left(1-\tfrac{41}{18}s_n\right) + O\left(\frac{T^{\frac{4}{3}}}{n^{\frac{11}{3}}}\right)\right], \tag{3.4a}$$

$$\begin{aligned}\omega_n = T^{-\frac{1}{2}}\Bigg[& r_n(1-s_n+s_n^2) - \frac{5\sqrt{3}}{12}s_n + \left(1-\frac{\sqrt{3}}{2}r_n s_n\right)\frac{\ln 3s_n}{3r_n} \\ & \pm i\left\{\sqrt{3}\, r_n s_n(1+s_n) - \frac{1}{2}\left(1+\frac{s_n}{3}\right)\ln 3s_n - \frac{5s_n}{12}\right\} + O\left[\left\{\frac{\ln [T^{\frac{1}{2}}/n]}{n}\right\}^2\right]\Bigg],\end{aligned} \tag{3.4b}$$

where $$r_n = (n+\tfrac{1}{3})\pi, \quad s_n = \left(\frac{T^{\frac{1}{2}}}{2r_n}\right)^{\frac{4}{3}}.$$

(iii) $n \ll T^{\frac{1}{2}}$, $T \gg 1$

$$\omega_0 = T^{-\frac{1}{4}}\left(1+\frac{19}{48T^{\frac{1}{2}}} + \frac{1343}{7680T} + O(T^{-\frac{3}{2}})\right), \tag{3.5a}$$

$$\begin{aligned}\omega_n = T^{-\frac{1}{6}}(2t_n)^{\frac{1}{3}}\Bigg[& 1 + \frac{\alpha}{3t_n^{\frac{2}{3}}} + \left(\tfrac{19}{18} - \frac{1}{81t_n^2}\right)\alpha^3 \\ & + \left(2t_n^{\frac{4}{3}} + \frac{19}{27t_n^{\frac{2}{3}}} + \frac{1}{243t_n^{\frac{8}{3}}}\right)\alpha^4 + O\left\{\left(\frac{n}{T^{\frac{1}{2}}}\right)^{\frac{8}{3}}\right\}\Bigg], \quad n \geqslant 1,\end{aligned} \tag{3.5b}$$

$$\omega_n = T^{-\frac{1}{6}}(2t_n)^{\frac{1}{3}}\left\{\left[1+\left(\tfrac{19}{18}-\frac{1}{81t_n^2}\right)\alpha^3\right]e^{\pm\pi i/3}\right.$$
$$\left.+\left[\frac{\alpha}{3t_n^{\frac{2}{3}}}+\left(2t_n^{\frac{4}{3}}+\frac{19}{27t_n^{\frac{2}{3}}}+\frac{1}{243t_n^{\frac{8}{3}}}\right)\alpha^4\right]e^{\mp\pi i/3}+O\left[\left(\frac{n}{T^{\frac{1}{2}}}\right)^{\frac{5}{3}}\right]\right\}, \quad n \geqslant 1, \qquad (3.5c)$$

where $$\alpha = (4T^{\frac{1}{2}})^{-\frac{1}{3}}, \quad t_n = n\pi.$$

Noting that the boundary-layer variable ξ in (3.2) is stretched by the factor $E^{\frac{1}{2}}$, see (2.7), one recovers, for $T = O(E^{-1})$, the well-known Stewartson $E^{\frac{1}{4}}$- and $E^{\frac{1}{3}}$-layer solutions from (3.2) and the lowest-order parts of (3.5*a*) and (3.5*b*, *c*), respectively. These expressions are thus higher-order extensions with respect to E of Stewartson's (1957) results and may be useful for calculation of Stewartson layers in cases where E is not exceedingly small.

Apart from the factor $T^{-\frac{1}{6}}$, which appears due to the scaling of the problem, the eigenvalues given by (3.3*a*, *b*) are, to lowest order, independent of T. This means that the eigenfunctions $(\hat{u}_{0n}, \hat{v}_{0n}, \hat{w}_{0n})$ corresponding to these eigenvalues describe motions, which are mainly governed by pressure and viscous forces and are only weakly affected by the Coriolis force. These eigenvalues do therefore have counterparts in similar boundary-layer problems where the Coriolis force is absent. The lowest-order part of (3.3*a*) is the same as that obtained from the almost trivial boundary-layer problem for the case when the plates are not rotating. In this case $\hat{v}_0 \neq 0$ and $\hat{u}_0 = \hat{w}_0 = 0$. The eigenvalues given by (3.3*b*) are the same as those appearing in certain problems involving the biharmonic equation† (see e.g. formulae (4.2)–(4.3) in Joseph & Sturges (1978)). For the geometry shown in figure 1, a problem of this kind would appear if, for example, the plates are not rotating and the flow is driven by a prescribed mass flux through the surface b, this mass flux being perpendicular to b and symmetric with respect to x_3^*. In this case one would have $\hat{v}_0 = 0$, $\hat{u}_0 \neq 0$ and $\hat{w}_0 \neq 0$.

The authors were unable to construct a useful (from the computational point of view) orthogonality relation for the eigenfunctions $\hat{\mathbf{u}}_{0n}$. This inconvenience arises frequently in similar problems governed by the biharmonic equation (see, for example, Spence 1977; Joseph & Sturges 1978). In the present work, a Galerkin method was used, i.e. the integral

$$\langle[\hat{v}_0+\mathbf{u}_0\cdot(\mathbf{k}\times\mathbf{e}_n)]^2+[\hat{u}_0+\mathbf{u}_0\cdot\mathbf{e}_n]^2+\hat{w}_0^2\rangle \qquad (3.6)$$

evaluated at $\xi = 0$, was minimized with respect to the coefficients c_n in the series

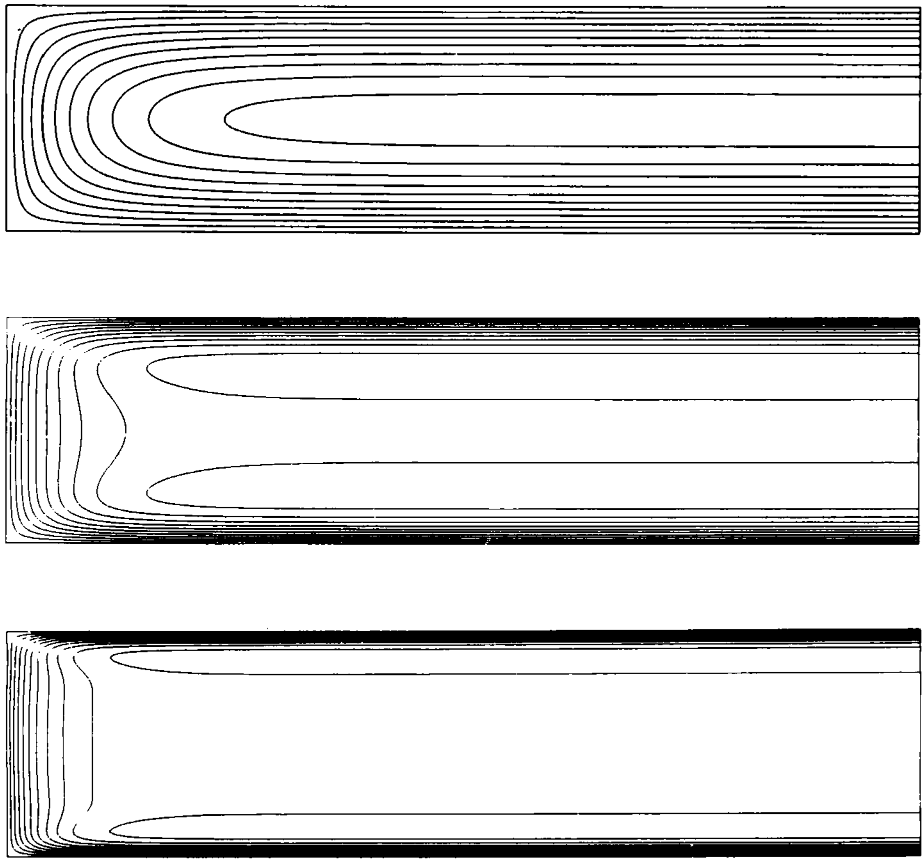
$$\hat{v}_0 = \sum_{n=0}^{N} c_n \sum_{K=1}^{3} C_{nK}e^{-\omega_n\xi}\cosh\nu_{nK}\,\zeta. \qquad (3.7)$$

For $N = 62$, the boundary conditions for the values of T in the examples in figures 4 and 5 are fulfilled with a relative error of order 10^{-3}, which was judged as satisfactory. This calculation, as well as an accurate calculation of the eigenvalues, was carried out numerically on a computer.

Figures 4 and 5 show level curves for the swirl velocity and streamlines for the meridional flow for three different values of the Taylor number.‡ The effect of rotation

† E.g. Stokes flow in trenches, deformation of elastic strips.

‡ The flow outside the boundary layer is in these graphs normalized such that $|\frac{1}{2}\,d\Pi_0/dz| = 1$, cf. (2.5).



(*a*)

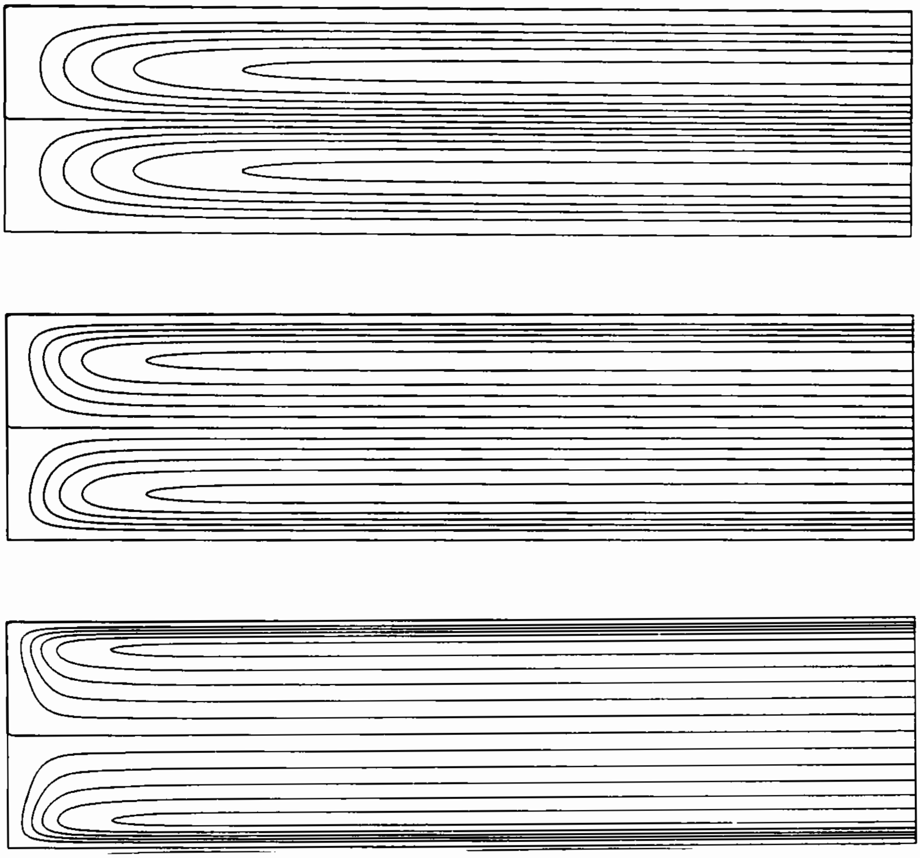
(*b*)

(*c*)

FIGURE 4. Level curves for the swirl velocity field, $|\zeta| \leqslant T^{\frac{1}{4}}$, $0 \leqslant \xi \leqslant 8T^{\frac{1}{4}}$.
(*a*) $T = 1$, $v = 0{\cdot}0735(j-1)$, $j = 1, \ldots, 11$;
(*b*) $T = 25$, $v = 0{\cdot}102(j-1)$, $j = 1, \ldots, 11$;
(*c*) $T = 100$, $v = 0{\cdot}102(j-1)$, $j = 1, \ldots, 11$.

is rather weak for the smallest value $T = 1$. The strength of the meridional flow is in this case only a few per cent of the swirl velocity. As in the non-rotating case, the horizontal extent of the vertical boundary layer is of the same order of magnitude as the distance between the plates. For the largest value of the Taylor number ($T = 100$), the flow outside the vertical boundary layer is characterized by two rather distinct Ekman layers separated by a thick region of geostrophic flow, where the velocity is essentially constant. Due to the gyroscopic constraint imposed on the flow by the relatively strong rotation in this case, the vertical boundary layer has in this case a thickness, which is significantly smaller than the distance between the plates (but larger than the thickness of the Ekman layers). The vertical boundary layer shown in figures 4(*c*) and 5(*c*) may be characterized as a short Stewartson $E^{\frac{1}{4}}$ layer. The eddies associated with the thinner Stewartson $E^{\frac{1}{3}}$ layer are weak in this case and can therefore not be seen on these graphs.

The same boundary-layer problem as the one considered in this work but for a rapidly rotating gas has been considered by Hashimoto & Matsuda (1979). These authors calculated the eigenvalues numerically for $T = \frac{1}{4}$ and used a collocation method



(a)

(b)

(c)

FIGURE 5. Streamlines for the meridional flow, $|\zeta| \leqslant T^{\frac{1}{4}}$, $0 \leqslant \xi \leqslant 8T^{\frac{1}{4}}$.
(a) $T = 1$, $\psi = \pm 0{\cdot}00139(j-1)$, $j = 1, \ldots, 6$.
(b) $T = 25$, $\psi = 0{\cdot}0586(j-1)$, $j = 1, \ldots, 6$.
(c) $T = 100$, $\psi = \pm 0{\cdot}0771(j-1)$, $j = 1, \ldots, 6$.

to fulfil the boundary conditions. Another similar problem has been considered by Jeyapalan & Bennet (1980), who showed that a solution of the form (3.7) agrees better with the experimental data by Baker (1967) for a free shear layer in a rotating fluid than does the usual Stewartson $E^{\frac{1}{3}}$-layer solution.

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